

# Quantum matter wave dynamics with moving mirrors

A. del Campo\*,<sup>1</sup> J. G. Muga,<sup>1</sup> and M. Kleber<sup>2</sup>

<sup>1</sup>*Departamento de Química-Física, UPV-EHU, Apartado 644, Bilbao, Spain*

<sup>2</sup>*Physik-Department T30, Technische Universität München,  
James-Frank Strasse, 85747 Garching, Germany*

When a stationary reflecting wall acting as a perfect mirror for an atomic beam with well defined incident velocity is suddenly removed, the density profile develops during the time evolution an oscillatory pattern known as diffraction in time. The interference fringes are suppressed or their visibility is diminished by several effects such as averaging over a distribution of incident velocities, apodization of the aperture function, atom-atom interactions, imperfect reflection or environmental noise. However, when the mirror moves with finite velocity along the direction of propagation of the beam, the visibility of the fringes is enhanced. For mirror velocities below beam velocity, as used for slowing down the beam, the matter wave splits into three regions separated by space-time points with classical analogues. For mirror velocities above beam velocity a visibility enhancement occurs without a classical counterpart. When the velocity of the beam approaches that of the mirror the density oscillations rise by a factor 1.8 over the stationary value.

PACS numbers: 03.75.Be, 03.75.-b, 03.75.Kk

## I. INTRODUCTION

Coherent and intense cold atom beams stand up for their applications in metrology, matter-wave interferometry or atom lithography [1]. A fast beam can be decelerated down to cold or ultracold temperatures by reflection from a mirror moving in the direction of the atoms. An early example is the achievement of an ultracold beam of neutrons by neutron reflection from a moving Ni-surface [2]. Moving mirrors for manipulating cold atom waves have been also produced with a time-modulated, blue-detuned evanescent light wave propagating along the surface of a glass prism [3, 4]. More recently bright beams of Helium atoms have been successfully slowed down using a moving Si-crystal on a spinning rotor [5]; and Rb atoms with a moving magnetic mirror on a conveyor belt [6], which provides a promising mechanism to generate a continuous, intense and very slow beam of guided atoms.

The motivation of the present work is that, whereas the analysis of the dynamics in such devices is usually based on classical mechanics, the long de Broglie wavelengths in the ultracold regime requires a quantum treatment. When operating in the quantum domain, such mechanisms are expected to exhibit a more subtle dynamics than simple classical trajectory reflection, with wave aspects becoming more prominent. Therefore our aim here is to study a simple solvable case which can be a reference for more complex settings. In this task we shall be guided by the abundant work on the “Moshinsky shutter”: One of the most relevant quantum transient effects in matter-wave beams is the diffraction in time effect [7, 8, 9, 10, 11], an oscillatory self-modulation of the density profile of a suddenly released beam. To date, it has been observed in a wide variety of systems

such as neutrons [12], ultracold atoms [13], electrons [14] and even Bose-Einstein condensates using vibrating mirrors [15]. However, the effect weakens with the width of the beam velocity distribution [16, 17], dissipation [18], environmental noise [11], finite-size of the beam and confinement [19, 20, 21, 22], strong interatomic interactions [23], and the smoothing of the aperture function of the shutter [16, 24, 25]. Indeed, any variant discussed with respect to the initial setup described by Moshinsky [8, 9] aimed to study the matter-wave beam dynamics, tends to wash out the oscillatory pattern of the beam profile (An exception is the long-time revival of the diffraction pattern described in [11].) In this paper, we examine the dynamics of a matter-wave beam in the presence of a moving mirror (see Fig. 1) and identify characteristic regimes and times, as well as quantum dynamical effects, such as the enhancement of the self-modulation of the beam profile. With an infinite velocity of the mirror, the usual diffraction in time result is recovered.

In section II, we review briefly the Moshinsky shutter problem and fix the notation; section III describes the diffraction in time for mirrors moving at finite velocity. The paper concludes with possible applications.

## II. DIFFRACTION IN TIME

We shall first consider a quasi-monochromatic atomic beam of momentum  $\hbar k$  (or velocity  $v_k = \hbar k/m$ ) impinging on a totally reflecting shutter at the origin  $x = 0$ ,

$$\psi_k(x', t' = 0) = 2i \sin(kx') \Theta(-x'), \quad (1)$$

where  $\Theta(x)$  is the Heaviside step function. Such state represents a standing matter-wave whose time evolution under sudden removal of the mirror at  $t = 0$  we shall denote by  $\psi_k^{(\infty)}(x, t)$ . Here the superscript  $(\infty)$  underlines the fact that the sudden removal of the mirror is equivalent to displacing it to the right with infinite velocity.

---

\*E-mail: adolfo.delcampo@ehu.es

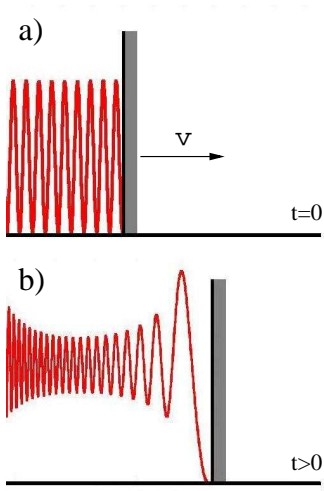


FIG. 1: Setup for a matter-wave beam reflected from a mirror moving with velocity  $v$ . a) At time equal to zero the beam forms a standing matter wave. b) During the time evolution, the density profile of the beam exhibits a self-modulation enhanced with respect to the sudden removal of the mirror, known as diffraction in time ( $v = \infty$ ).

This wave is clearly not normalized, but it can be considered as an elementary component of a semi-infinite wave packet. (Following customary practice, we shall liberally speak of its square modulus as a “density” even if it is dimensionless.) Since for  $t > 0$  the dynamics is free, it can be obtained using the superposition principle

$$\psi_k^{(\infty)}(x, t) = \int_{-\infty}^{\infty} dx' K_0(x, t|x', t' = 0) \psi(x', t' = 0) \quad (2)$$

with the free propagator

$$K_0(x, t|x', t') = \left[ \frac{m}{2\pi i \hbar (t - t')} \right]^{\frac{1}{2}} e^{i \frac{m(x - x')^2}{2\hbar(t - t')}}. \quad (3)$$

In a seminal paper [8], Moshinsky proved that

$$\psi_k^{(\infty)}(x, t) = M(x, k, t) - M(x, -k, t), \quad (4)$$

where, the so-called Moshinsky function [8, 9]

$$M(x, k, t) := \frac{e^{i \frac{mx^2}{2\hbar t}}}{2} w(-z), \quad z = \frac{1+i}{2} \sqrt{\frac{\hbar t}{m}} \left( k - \frac{mx}{\hbar t} \right), \quad (5)$$

is related to the Faddeyeva function  $w(z) := e^{-z^2} \text{erfc}(-iz)$  [26, 27]. On physical grounds it is clear that each of the  $M$  functions corresponds to a freely time-evolved cut-off plane-wave. Such solution entails the well-known diffraction in time phenomenon which consists in a set of oscillations in the beam profile, in dramatic contrast with the classical case which is simply described by  $\Theta(v_k t - x)$  for  $x > 0$ . [In principle a more complex “classical analog” may be established using the Wigner function and classical trajectories with negative weights

[7, 28], but in this work the “classical case” refers always to the incident beam formed by classical particles with fixed velocity  $v_k$ .]

The asymptotics of the Moshinsky functions for  $|x - \hbar k t / m| \rightarrow \infty$  can be found from those of  $w(z)$  when  $z \rightarrow \infty$  [17]. In the classical region (where the classical beam density is non-zero),  $x \leq v_k t$ , one finds in terms of the Gamma function  $\Gamma(y)$  that

$$M(x, k, t) \sim e^{ikx - i \frac{\hbar k^2 t}{2m}} + \frac{e^{i \frac{mx^2}{2\hbar t}}}{2\pi i} \sum_{n=0}^{\infty} \frac{\Gamma(n + \frac{1}{2})}{z^{2n+1}}, \quad (6)$$

whereas in the complementary region  $x > v_k t$  only the series survives. Therefore the classical front plays an essential role in the quantum dynamics.

It was pointed out in [16] that the initial state given by Eq. (1), an eigenstate of a totally reflecting mirror, maximizes the diffraction in time pattern with respect to the different types of hard-wall mirrors. Moreover, the problem can be reformulated as a sudden turn-on of a matter-wave source of the form  $\sigma(x, t) \propto \delta(x)\Theta(t)$  (an inhomogeneous term, in the time-dependent Schrödinger equation) [29].

### III. MOVING MIRROR WITH FINITE VELOCITY

In this section we study the diffraction in time phenomenon when the reflecting wall, acting as a perfect mirror, is displaced to the right with finite velocity,  $x_m = vt$ , for  $t > 0$ . It is useful to examine first the effect of the moving mirror for a classical beam of particles with incident velocity  $v_k$ . Two different regimes are possible depending on whether the velocity of the mirror  $v$  is larger or smaller than  $v_k$ : If  $v > v_k$  no particle could hit the mirror for  $t > 0$  so there is no effect of the moving mirror in the beam dynamics. This case is therefore classically equivalent to the sudden removal of the mirror; The second case,  $v < v_k$ , is instead characterized by the occurrence of reflection. The particles reflected by the moving mirror have velocity  $2v - v_k$  (since  $v - v_k$  is their velocity in the reference frame moving with the mirror), and their front (marked by the first particles reflected just after the mirror starts moving, at  $t = 0_+ = \epsilon > 0$ , when  $\epsilon \rightarrow 0$ ) is at  $x_+ = (2v - v_k)t$ . This critical point may move to the right or left depending on the sign of  $2v - v_k$ . A second critical space-time point at  $x_- = -v_k t$  corresponds to the advancement leftwards of the last particle reflected with the stationary mirror at  $t = 0_-$ . To the left of  $x_-$  the beam is composed by particles incoming and reflected with velocities  $\pm v_k$ , so that the effect of the moving mirror has not arrived yet. In the intermediate region  $x_- < x < x_+$ , only right-moving particles are found with velocity  $v_k$ . Finally, for  $x > x_+$  there are incident particles and reflected ones with velocities  $v_k$  and  $2v - v_k$  respectively.

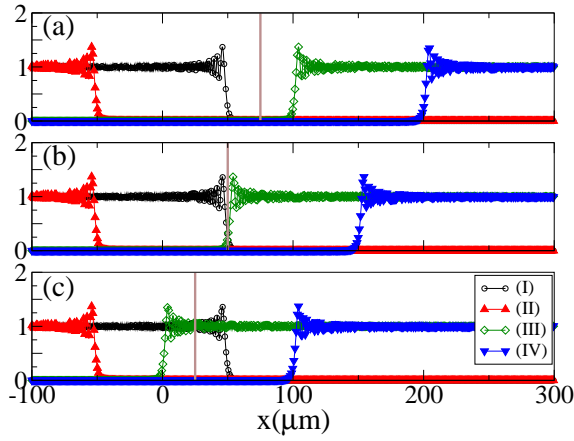


FIG. 2: Density profile of the components involved in the quantum dynamics in the presence of a mirror. The snapshots are taken for a  $^{87}\text{Rb}$  atom beam moving at  $v_k = 1.0$  cm/s, impinging on a mirror which has been displaced for 5 ms at a)  $v = 1.5$ , b) 1.0, and c) 0.5 cm/s. The position of the mirror is denoted by a brown vertical line. The components travel with  $v_k$  (I) and  $-v_k$  (II) as if in the absence of the mirror, and the corresponding images that warrant the boundary condition at  $x_m = vt$ , travel with wavefronts at  $(2v - v_k)t$  (III) and  $(2v + v_k)t$  (IV). Since (IV) moves into the forbidden region, it soon stops playing a physical role.

To cope with a moving mirror in quantum mechanics, we take advantage of the result for the propagator obtained with semiclassical arguments in [30] and alternatively by summing  $\delta$ -function perturbation series in path integrals [31], which turns out to be exact. Its explicit form is

$$K_w^{(v)}(x, t|x', t') = e^{-i\frac{m}{\hbar}\left[v(x-vt)-v(x'-vt')+v\frac{(t-t')^2}{2}\right]} \times [K_0(x-vt, t|x'-vt', t') - K_0(x-vt, t|-x'+vt', t')]. \quad (7)$$

It is of the “collapsed” kind [32], since it is obtained from just two classical paths.

Let us assume the Moshinsky initial condition (1) of a quasi-monochromatic beam incident on the shutter, which is located at the origin at time  $t' = 0$ . At a later time the wavefunction can be calculated using the integral equation (2) with the kernel of Eq.(7) and the initial state  $\psi(x', t' = 0) = 2i \sin(kx')\Theta(-x')$ .

Performing the integrals, one finds

$$\begin{aligned} \psi_k^{(v)}(x, t) = & e^{i\frac{mv}{\hbar}x - i\frac{mv^2}{2\hbar}t} \\ & \times \left[ M\left(x - vt, k - \frac{mv}{\hbar}, t\right) - M\left(x - vt, -k - \frac{mv}{\hbar}, t\right) \right. \\ & \left. - M\left(vt - x, k - \frac{mv}{\hbar}, t\right) + M\left(vt - x, -k - \frac{mv}{\hbar}, t\right) \right], \end{aligned} \quad (8)$$

for the physical region  $x \leq x_m$ . The physical wave function is of course zero in the forbidden region but it is

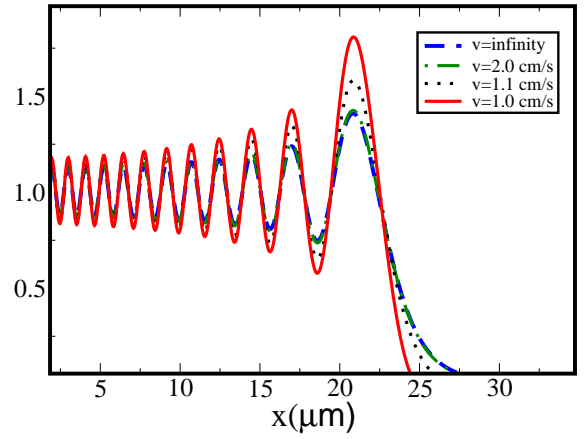


FIG. 3: Characteristic modulation of the density profile of a matter-wave beam of  $^{87}\text{Rb}$  atoms induced by a totally reflecting mirror which is displaced at constant velocity  $v > v_k$  ( $v_k = 1.0$  cm/s). The oscillatory pattern is maximized as  $v$  assumes the velocity of the beam. The case  $v \rightarrow \infty$  reduces to the diffraction in time setup with the lowest fringe visibility.

useful to consider formally Eq. (8) also at  $x > x_m$  for the simple analysis of “image” points and term contributions. The first two terms,  $M_I$  and  $M_{II}$  for short, describe the free time evolution of the beam in the absence of the mirror,  $\psi^{(\infty)}(x, t)$ , with wavefronts at  $x = \pm v_k t$  (note that  $v_k t$  is in the forbidden region when  $v_k > v$ ); whereas the third and fourth terms,  $M_{III}$  and  $M_{IV}$  for short, are their corresponding images with respect to the position of the mirror  $x_m$ , and are relevant for  $x \gtrsim (2v \pm v_k)t$ . The corresponding “densities” are shown in Fig. 2. Clearly  $M_{IV}$ , whose wavefront is at  $(2v + v_k)t$  represents in general a minor contribution for it travels into the forbidden region beyond  $x_m$  for all  $v$ , whereas the front of  $M_{III}$ , at  $x_+ = (2v - v_k)t$  enters into the forbidden region for  $v > v_k$ .

In a reference frame moving with the mirror, the dynamics arises as a result of a kick imparted on the beam. Let us check first the consistency of the mathematical result in the limits of very slow and very fast walls. For a very slow wall,  $v \sim 0$ , the result reduces to that of a fixed wall, with  $\psi_k^{(v)}(x, t) \sim \psi_k(x, t)$ , as it follows using the exact relation

$$M(x, k, t) + M(-x, -k, t) = e^{ikx - i\frac{\hbar k^2}{2m}t}. \quad (9)$$

( $M_I + M_{IV}$  give an incident plane wave and  $M_{II} + M_{III}$  the reflected wave with a global minus sign to produce the sine in Eq. (1).) In the opposite case where the velocity of the moving wall is much bigger than that of the incident beam the two last terms in Eq. (8) become negligible and one has  $\psi_k^{(v)}(x, t) \sim M(x, k, t) - M(x, -k, t) = \psi_k^{(\infty)}(x, t)$ . This is the limit of infinitely fast removal of the shutter, which lead to the discovery of diffraction in time.

As for the classical beams, two different velocity regimes can be clearly distinguished; as expected, in both

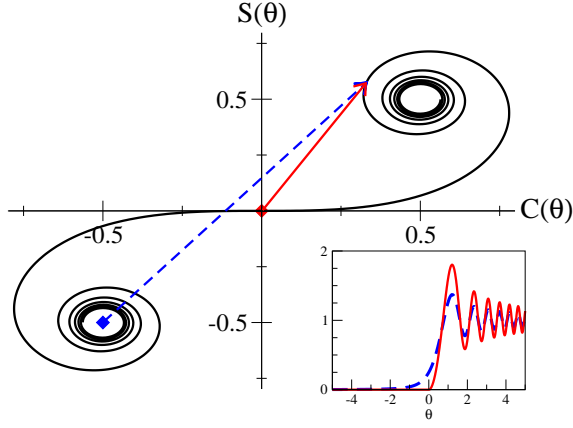


FIG. 4: Universal representation of the beam profile for ordinary (dashed line) and enhanced (solid line) diffraction in time in terms of the Cornu spiral, when the velocity of the mirror approaches that of the beam. The inset shows the probability density of both beam profiles as a function of the variable  $\theta$ .

velocity regimes the quantum wave shows characteristic diffraction in time patterns. When the velocity of the mirror is larger than the mean velocity of the beam nothing happens classically different from  $v = \infty$  because of the absence of reflection, but quantally the amplitude of the oscillations increases whereas its spacing is unaffected. Figure 3 shows that the presence of the mirror leads to an enhancement of the diffraction in time as the velocity of the wall approaches the mean velocity of the beam. The maximum intensity  $P_{max}$  of the main peak increases as  $v \rightarrow v_k$  reaching an upper bound which is 1.816 times the stationary value, whereas in the Moshinsky solution, Eq.(4), the increment is limited to a 1.37 times the stationary value [33]. Therefore the dominant effect is the enhancement of diffraction in time pattern. This can be understood as follows: As  $v \rightarrow v_k$  and for  $0 < x < x_m$  we can neglect the second and fourth terms in Eq. (8),

$$\psi_k^{(v)}(x, t) \simeq e^{i\frac{mv_x}{\hbar} - i\frac{mv^2 t}{2\hbar}} \times [M(x - vt, 0, t) - M(vt - x, 0, t)]. \quad (10)$$

Using Eq. (9) together with  $M(x, k, t) = e^{ikx - i\frac{\hbar k^2 t}{2m}} \text{erfc}[\frac{1+i}{2}\sqrt{\hbar t/m}(k - mx/\hbar t)]$ , and  $\text{erfc}(z) + \text{erfc}(z) = 1$ , this may be written as

$$\psi_k^{(v)}(x, t) \simeq e^{i\frac{2mv_x}{\hbar} - i\frac{mv^2 t}{\hbar}} \text{erfc}\left[\frac{1+i}{2}\sqrt{\frac{\hbar t}{m}}\left(\frac{mv}{\hbar} - \frac{mx}{\hbar t}\right)\right]. \quad (11)$$

Its absolute square value admits a simple geometric interpretation in terms of the Cornu spiral or clothoid, which is the curve that results from a parametric representation of the Fresnel integrals,  $S(\theta)$  versus  $C(\theta)$  as shown in Fig. 4.

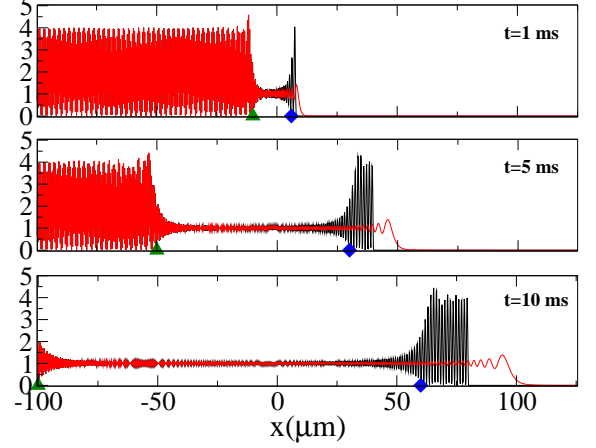


FIG. 5: Transients in the density profile  $|\psi_v(x, t)|^2$  of an atom beam when the velocity of the mirror is less than the mean of the beam (solid line) versus the sudden-removal case (dashed line), with  $t = 10$  ms,  $v = 0.8$  cm/s and  $v_k = 1.0$  cm/s. The triangles point out the classical position of a particle moving with  $-v_k$ , and the diamonds the location of a particle after bouncing off from the mirror,  $(2v - v_k)t$ . The oscillations of the matter-wave front reach a visibility  $V = 1$  tending to the asymptotic form.

Introducing  $\theta = \sqrt{\hbar t/m\pi}(\hbar kt/m - x)$ , the universal representation of the beam profile reads

$$|\psi_k^{(v)}(x, t)|^2 \simeq 2\{[S(\theta)]^2 + [C(\theta)]^2\}, \quad (12)$$

this is, twice the distance from the origin (where  $\theta = 0$ , at the position of the mirror) to any point of the spiral with  $\theta > 0$  (first quadrant), being zero elsewhere [34]. Moshinsky [8] showed that whenever the  $-k$  component in the ordinary diffraction in time problem, described by  $\psi_k^{(\infty)}(x, t)$ , can be neglected, the beam profile admits also a universal representation in the form,

$$|\psi_k^{(\infty)}(x, t)|^2 \simeq \frac{1}{2}\left\{\left[S(\theta) + \frac{1}{2}\right]^2 + \left[C(\theta) + \frac{1}{2}\right]^2\right\}, \quad (13)$$

see Fig.4, which is half the distance from the point  $(-1/2, -1/2)$  to the Cornu spiral for arbitrary  $\theta$ . It follows that the frequency of the oscillations is the same for the ordinary and enhanced diffraction in time in the limit  $v \rightarrow v_k^+$ . Moreover the width of the fringes  $\delta x$  is also common to both cases, and can be estimated from the intersection between the classical and quantum probability densities [8, 16], leading to a dependence of the form

$$\delta x \propto (\pi\hbar t/m)^{1/2}. \quad (14)$$

Note that the intensity of the beam tends to unity in both cases for  $\theta \rightarrow \infty$  (i.e., away from the mirror).

From a more physical perspective the enhancement is due to the contribution of the image  $M(vt - x, 0, t)$  of

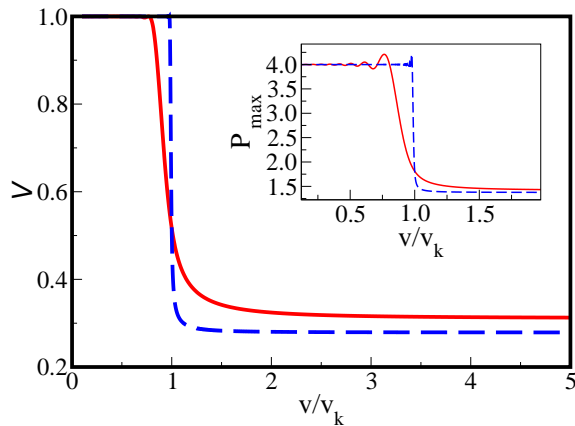


FIG. 6: Visibility of the fringes in the density profile of an atom beam for  $v_k = 0.1$  (solid line), and  $1.0$  cm/s (dashed line) at time  $t = 100$  ms. The inset shows the maximum of the main fringe for the same range of  $v/v_k$  values.

the main term  $M(x - vt, 0, t)$ , i.e., to a quantum reflection contribution (see Fig. 2b). In the classical limit, the trajectory closest to the mirror remains unaffected by the presence of the mirror if  $v \geq v_k$ ; however in quantum mechanics the front of  $M(x - vt, 0, t)$  is not sharply localized and in addition some reflection occurs. This effect becomes smaller when  $v - v_k$  increases because of the displacement of the image term front into the forbidden region; equivalently, the front tail of the leading Moshinsky function lags behind the mirror. As a result no reflection will occur, as demonstrated in Fig. 2a.

In the opposite regime, if  $v_k > v$ , the wavefront at  $x_+$  bounces off from the mirror leading to an interference enhancement and the progressive construction of the new scattering state with velocities  $v_k$  (incident) and  $2v - v_k$  (reflected) in the domain  $x_+ < x < vt$ . The dynamical evolution is illustrated in Figure 5: as a consequence of the reflection, the interference pattern can reach values up to four times the stationary one. Indeed, during the evolution of the beam three distinct regions can be identified,  $|\psi_k^{(v)}(x, t)|^2 \sim$

$$\begin{cases} 4 \sin(kx) & , \quad x \leq x_- \\ 1 & , \quad x_- < x \leq x_+ \\ 4 \sin[(k - v)(x - vt)] & , \quad x_+ < x < vt \end{cases} \quad (15)$$

where, the separating limits are not sharp but described by the corresponding Moshinsky functions.

To characterize the amplitude of the oscillations we may consider the visibility of the main fringe, defined by

$$V = \frac{P_{max} - P_{min}}{P_{max} + P_{min}}, \quad (16)$$

where  $P_{min}$  is the first minimum of the wave density behind the matter-wave front. Figure 6 shows how for a fixed time, this measure exhibits a monotonic decay with increasing ratio  $v/v_k$ , reaching the sudden-removal result as  $v/v_k \rightarrow \infty$ .

#### IV. CONCLUSION AND DISCUSSION

A standing matter wave suddenly released, develops during its propagation an oscillatory pattern in the density profile, a phenomenon known as diffraction in time. We have shown that if the beam is released by moving the mirror at finite velocity there is an enhancement of such effect and of the visibility of the corresponding fringes. The result is relevant for recently proposed atom beam techniques, such as beam slowing with mirrors on spinning rotors [5] or conveyor belts [6], and schemes for atom interferometry in time domain when operating with ultracold velocities. Being intrinsically a matter-wave effect, the enhancement of the diffraction in time could be observed with ultracold neutrons as well [35].

Further extension of this work can be envisaged to deal with the dynamics of finite pulses [9, 11, 16], and the use of time dependent external fields for controlled transport of matter waves [36, 37].

#### Acknowledgments

This work has been supported by DGES-Spain (FIS2006-10268-C03-01) and UPV-EHU (grant 00039.310-15968/2004). AC acknowledges discussions with M. G. Raizen and S. Godoy, the hospitality of M. Kleber and H. Friedrich's group at the Physik-Department of the Technische Universität München where part of this work was carried out, and a fellowship from the Basque Government (BFI04.479)

- 
- [1] P. Berman, *Atom interferometry*, (San Diego, Academic 1997).
  - [2] A. Steyerl, H. Nagel, F. X. Schreiber, K. A. Steinhauser, R. Gähler, W. Gläser, P. Ageron, J. M. Astruc, W. Drexel, G. Gervais, W. Mampe, *Phys. Lett. A* **116**, 347 (1986).
  - [3] A. Steane, P. Szriftgiser, P. Desbiolles, J. Dalibard, *Phys. Rev. Lett.* **74**, 4972 (1995)
  - [4] M. Arndt, P. Szriftgiser, J. Dalibard, A.M. Steane, *Phys. Rev. A* **53**, 3369 (1996).
  - [5] A. Libson *et al.*, *New. J. Phys.* **8**, 77 (2006).
  - [6] G. Reinaudi *et al.*, *Eur. Phys. J. D.* **40**, 405 (2006).
  - [7] V. I. Man'ko, M. Moshinsky, and A. Sharma, *Phys. Rev. A* **59**, 1809 (1999).
  - [8] M. Moshinsky, *Phys. Rev.* **88**, 625 (1952).
  - [9] M. Moshinsky, *Am. Jour. Phys.* **44**, 1037 (1976).
  - [10] M. Kleber, *Phys. Rep.* **236**, 331 (1994).
  - [11] A. del Campo, J. G. Muga, and M. Moshinsky, *J. Phys. B* **40**, 975 (2007).
  - [12] T. Hils, J. Felber, R. Gähler, W. Gläser, R. Golub, K. Habicht, and P. Wille, *Phys. Rev. A* **58**, 4784 (1998).
  - [13] A. Steane, P. Szriftgiser, P. Desbiolles, and J. Dalibard,

- Phys. Rev. Lett. **74**, 4972 (1995); M. Arndt, P. Szriftgiser, J. Dalibard, and A. M. Steane, Phys. Rev. A **53**, 3369 (1996); Phys. Rev. A **53**, 3369 (1996).
- [14] F. Lindner *et al.*, Phys. Rev. Lett. **95**, 040401 (2005).
- [15] Y. Colombe, B. Mercier, H. Perrin, and V. Lorent, Phys. Rev. A **72**, 061601(R) (2005).
- [16] A. del Campo and J. G. Muga, J. Phys. A: Math. Gen. **38**, 9803 (2005).
- [17] J. G. Muga and M. Büttiker, Phys. Rev. A, **62**, 023808 (2000).
- [18] M. Moshinsky and D. Schuch, J. Phys. A: Math. Gen. **34**, 4217 (2001).
- [19] A. S. Gerasimov and M. V. and Kazarnovskii, Sov. Phys. JETP **44**, 892 (1976).
- [20] S. Godoy, Phys. Rev. A **65**, 042111 (2002).
- [21] S. Godoy, Phys. Rev. A **67**, 012102 (2003).
- [22] E. Granot and A. Marchewka, Phys. Rev. A **76**, 012708 (2007).
- [23] A. del Campo, J. G. Muga, Europhys. Lett. **74**, 965 (2006).
- [24] G. Scheitler and M. Kleber, Z. Phys. D **9**, 267 (1988).
- [25] E. Granot and A. Marchewka Europhys. Lett. **72**, 341 (2006).
- [26] V. N. Faddeyeva and N. M. Terentev, *Mathematical Tables: Tables of the values of the function  $w(z)$  for complex argument* (Pergamon, New York, 1961).
- [27] A. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965)
- [28] J. G. Muga, R. Sala, R. F. Snider, Phys. Scripta **47**, 732 (1993).
- [29] M. Moshinsky, Rev. Mex. Fis. **1**, 151 (1952).
- [30] M.G.E. da Luz and B.K. Cheng J. Phys. A: Math. Gen. **25**, L1043 (1992).
- [31] C. Grosche, Ann. Phys. **6**, 557 (1993).
- [32] R.E. Crandall, J. Phys. A: Math. Gen. **16**, 513 (1983).
- [33] We have also considered the case of a semi-transparent mirror of the form  $V(x) = \eta\delta(x - vt)$ . The resulting dynamics is still exactly solvable, and shows that the enhancement of the diffraction is maximized in the limit  $\eta \rightarrow \infty$ , discussed here.
- [34] We use the relation  $C(z) + iS(z) = \frac{1+i}{2}[1 - \operatorname{erfc}(\sqrt{\pi}(1 - i)z/2)]$  [27].
- [35] S.N. Balashov *et al.* Physica B **350**, 246 (2004).
- [36] A. del Campo and J. G. Muga, J. Phys. A: Math. Gen. **39**, 5897 (2006).
- [37] Y.D. Song, Europhys. Lett. **65**, 622 (2003)